CABLE SCISSORS ARCH - MARIONETTIC STRUCTURE

Kokawa Tsutomu, Hokkaido Tokai University, Japan

Abstract

Cable Scissors Arch(CSA) has a possibility to introduce a dynamical expression into an architectural space design,because it can easily change the geometry all the time with keeping stability based on the following simple idea - winding up or back a zigzag cable passing through the pulleys installed at the connection points between the scissors units. The structural form endlessly changes in time and space without the beginning and ending,and its dynamical situations will give a thrilling and exciting atmosphere. As a kind of string such as cable is very important in this structure, CSA may be called "Marionettic Structure" named after Marionette. CSA will be useful for not only the improvement of construction method but also enjoying the changing structural form itself.
This paper mainly describes 1) a trial construction of 11m span timber model and 2) theoretical analysis of statics under changing geometry and 3) consideration on winch tension force between the experiment and the analysis.

1. Introduction

Modern structures with changing geometries such as Pantadome(Kawaguchi [1]) or Scissors Structure(Escrig et al. [2]),have been utilized for a rational construction technique in architectural roofs. Cable Scissors Arch (CSA) will be also used for the same purpose. In addition to such an application,it has a possibility of introducing directly a dynamical expression into the architectural space design,because CSA can continuously change the geometry with holding stability. The structural form endlessly changes in time and space without the beginning and ending,and its dynamical suituations will give a srilling and exciting atmosphere to us. The structural idea basically comes from scissors mechanism. CSA consists of three-hinged arch scissors and zigzag flexible cables through pulleys installed at the connection points between the scissors units,as shown in Fig.1(Kokawa [3]). During winding up the cable by a winch, CSA is going to expand and be forced to lift up as a result. On the other side,it will be shortened and go down by its self-weight during the winding back. The weight of the structure is in equilibrium with a compression in the strut and a tension in the cable,through the whole operation. As a tension force always works in the cable under gravity load, CSA is a stable and statically determinate structure all the time. This structure might be considered as a kind of truss structure which resists mainly axial forces although without the chords, and it is numerically confirmed CSA has far the better mechanical efficiency in the points of strength and rigidity,compared with 'Scissors Arch without Cable' (Kokawa [4]). Above all, the zigzag cable in CSA plays important roles for not only a simple devise of changing geometry but also an improvement of structural efficiency by truss action.
This paper mainly describes 3 items. The first one is the construction test on a 11-m span timber spatial arch model with changing geometry from 175cm to 500cm in rise. The second is concerned with the structural analysis under changing geometry based on a mathematical treatment of the pulley joints as 'Slipping Pin Joint with Friction'. And the third describes some consideration on the comparison between the experimental winch tension force and the numerical analysis.

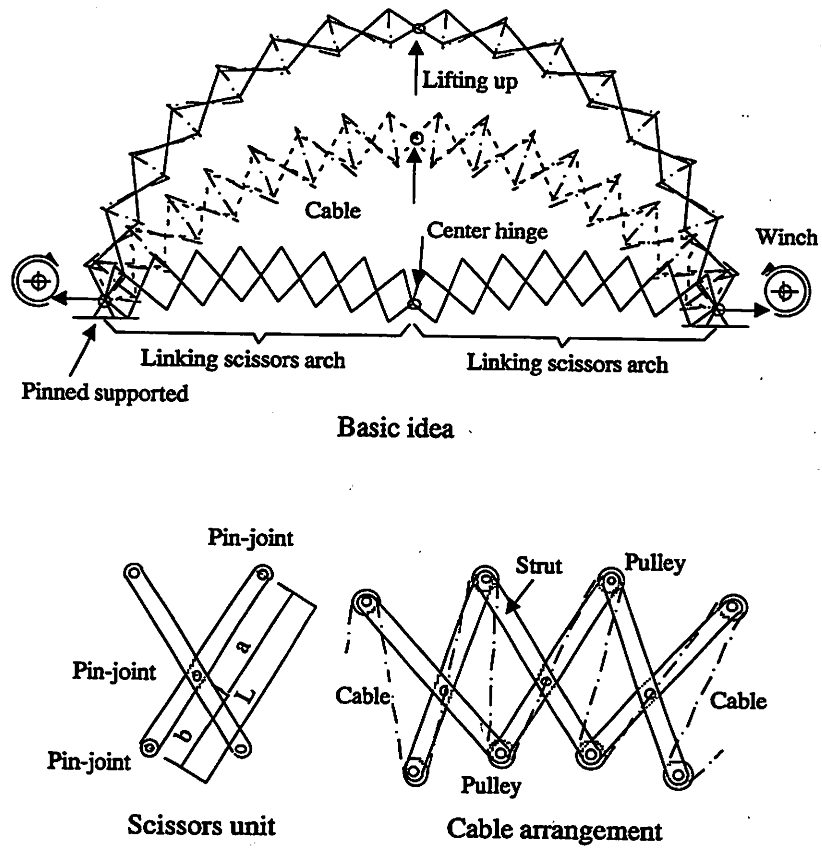

Fig.1.Structural principle of CSA

2. Construction test

In order to investigate on the validity of the structural idea,experimental construction studies on some models were carried out (Kokawa [3]), and the construction details have been improving a bit by bit. Fig.2 shows several stages of a latest experimental model with sequential changing geometries under winding up. The model has 11-m span, the number of scissors units is 8 in half side, the strut length is 115cm(a=60cm,b=55cm) and the general form of the model is a cylindrical roof type. In order to prevent the arch from the excessive lateral deformation,three CSAs with V-section are located in parallel with each other,and connected to longitudinal horizontal members as shown in Fig.3. A kind of furniture fittings and ball bearings are used for the joint-plates and the pulleys, respectively. Membrane as the roof material,is in advance fixed slackly to the longitudinal horizontal members. It will be stretched at the desired geometry. The linking scissors are made of wood and the section of the strut is 2cm*4cm. On the other side,the diameter of the cable is 3mm. The point of central hinge moves almost vertically because the length change of cables is same at both left and right side. All cables are finally connected to a hand winch in this test. Total

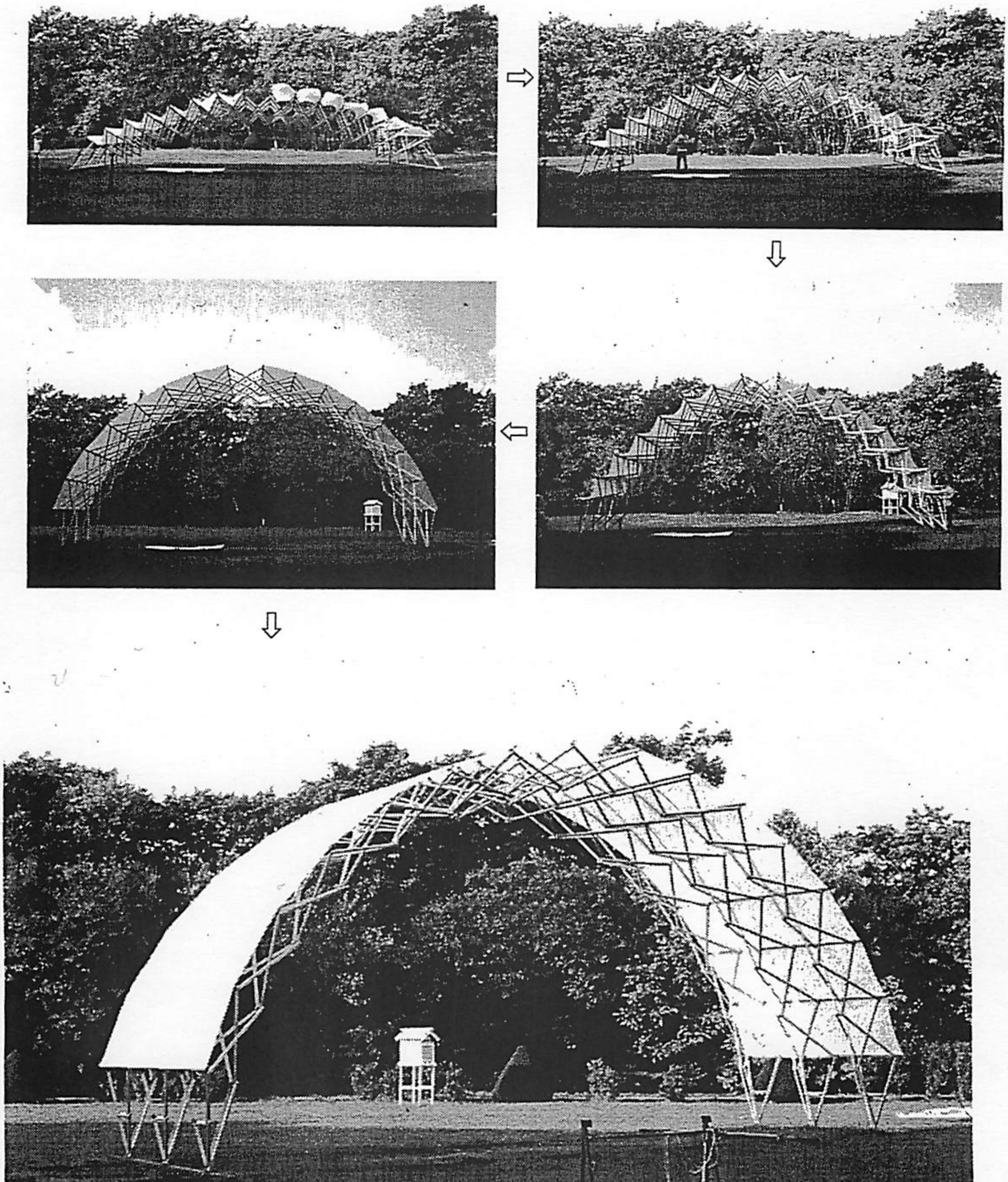

Fig.2.Sequential changing geometry

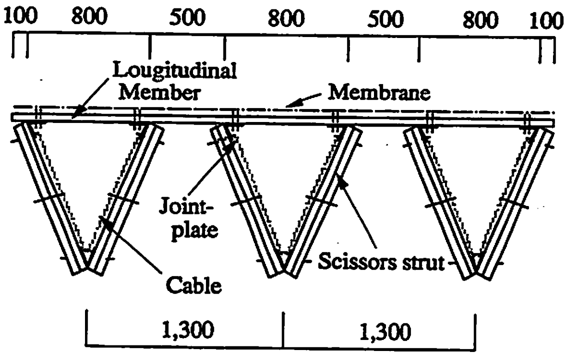

Fig.3.Model section

weight of this model is about about 190kg. In this test,the relation between the rise: h from 150cm to 500cm and the winch tension force is obtained experimentally several times by a data logger. It is qualitatively observed the winch tension force is inversely proportional to h. In spite of the operation of winding up and back by a hand winch,the general geometry of the structure smoothly and continuously changes.Besides this model,another type of model without membrane was also examined.

3. Winch force analysis

Fig.4 draws some stages of the changing geometry based upon a numerical method. At the previous paper,the statical analysis of the final stage was developed by a mathematical treatment of the pulley-joint as 'Slipping Pin Joint without Friction between Cable and Pulley' (Kokawa[4]). However, CSA is able to change the geometry all the time, so it is very important to predict numerically how the structural behaviour up to the final stage changes. Improving the previous method better,the statical analysis of the arbitrary stage is briefly developed here, based on a mathematical treatment of the pulley-joint as 'Slipping Pin Joint with Friction'. And then,some considerations on the experimental winch force at the above-mentioned model will be discussed, based on the following numerical analysis.

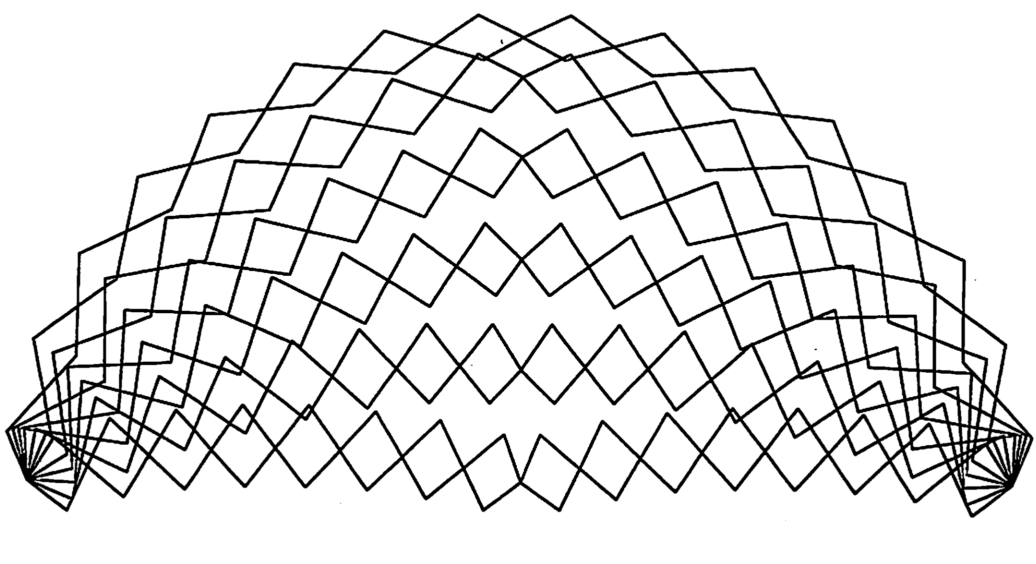

Fig.4. Computed sequential changing geometries

3.1. Outline of Statical Analysis under changing geometry

In this analysis,each cable starts at the central hinged point and zigzag passes through pulleys to each supporting end as shown in Fig.1. If the length variations of cable at both side under winding up or back,are same,the center point will be constrained horizontally and move vertically. As small friction forces exist between the cable and the pulley joint,the mathematical model of the joint is simply treated here as 'Slipping Pin Joint with Friction between them' obeyed by a rule as shown in Fig.5. Therefore,the distribution of the cable-tension is decided according to the winding up or back. As this structure is a kind of stable and statically determinate structure,only the equilibrium conditions are needed to find out the stress of members. Fig.6 shows all kinds of forces in unit [i] and [i+1]. Considering the equilibrium(ΣX=0, ΣY=0) at joint no. @ and A,the relation between the right side forces vector: {Xir} and the left side vector: {Xil},is given in Eq.(1).

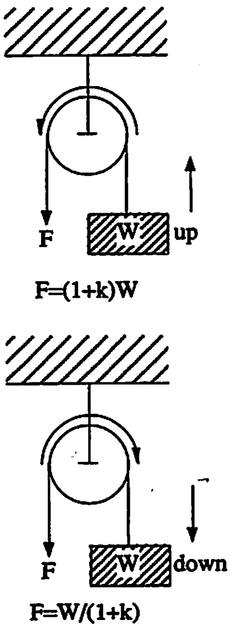

Fig.5. Friction model between pulley and cable

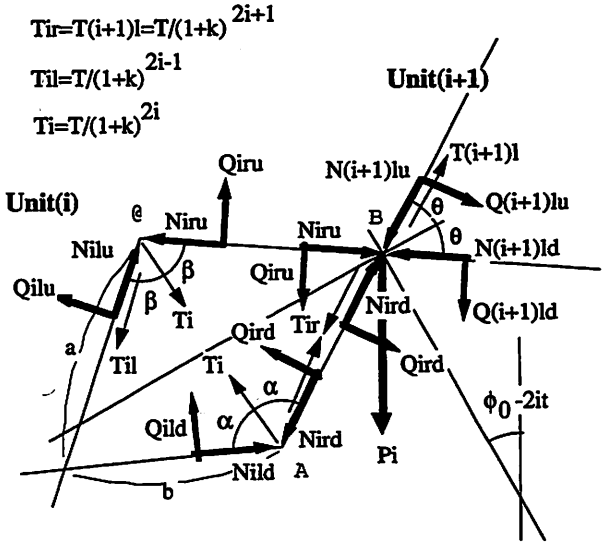

Fig.6. Internal and external forces (Winding up)

$$\{X_{ir}\}=[A]\{X_{il}\}+\{a_i\}T \qquad (1)$$

Where,

$$\{X_{ir(1)}\}=\begin{Bmatrix} N_{ir(1)u} \\ Q_{ir(1)u} \\ N_{ir(1)d} \\ Q_{ir(1)d} \end{Bmatrix} \qquad \{a_i\}=\begin{Bmatrix} (\cos\beta+(1+k)\cos2\beta)/(1+k)^{2i} \\ (\sin\beta+(1+k)\sin2\beta)/(1+k)^{2i} \\ (\cos\alpha+1/(1+k))/(1+k)^{2i} \\ -\sin\alpha/(1+k)^{2i} \end{Bmatrix}$$

$$[A]=\begin{bmatrix} -\cos2\beta & -\sin2\beta & 0 & 0 \\ -\sin2\beta & \cos2\beta & 0 & 0 \\ 0 & 0 & -\cos2\alpha & \sin2\alpha \\ 0 & 0 & \sin2\alpha & \cos2\alpha \end{bmatrix}$$

and T denotes the unknown cable tension force at the end.And,the relation between {Xir} and {X(i+1)l} is shown in Eq.(2), considering the equilibrium of point no. B.

$$\{X_{(i+1)l}\}=[B]\{X_{ir}\}+\{b_i\}P_i \qquad (2)$$

$$[B]=\begin{bmatrix} 0 & -(1+\xi)\mathrm{cosec}2\theta & 1 & -(1+1/\xi)\cot2\theta \\ 0 & 0 & 0 & 1/\xi \\ 1 & (1+\xi)\cot2\theta & 0 & (1+1/\xi)\mathrm{cosec}2\theta \\ 0 & \xi & 0 & 0 \end{bmatrix}, \qquad \xi=a/b$$

$$\{b_i\}=\begin{Bmatrix} -\cos(\phi_0-2it-\theta)/\sin2\theta \\ 0 \\ \cos(\phi_0-2it+\theta)/\sin2\theta \\ 0 \end{Bmatrix}$$

Eq.(3) which indicates the relation between {Xil} and {X(i+1)l}, is derived from Eqs.(1) and (2).

$$\{X_{(i+1)l}\}=[C]\{X_{il}\}+\{d_i\}T+\{b_i\}P_i \qquad (3)$$

Where, [C]=[B][A] and { di }=[B]{ ai }
So, {Xil} can be expressed by using {X1l} which indicates the end force vector connected to the pin support,and then {Xnl} is obtained by substituting i=n into the equation. Finally, the other end force vector:{Xnr} which is connected to the central hinged point,is expressed in Eq.(4)

$$\{X_{nr}\}=[A][C]^{(n-1)}\{X_{1l}\}+[A]\left(\sum_{k=1}^{n-1}[C]^{(n-k-1)}\right)\{d_k\}T$$
$$+[A]\sum_{k=1}^{n-1}[C]^{(n-k-1)}\{b_k\}P_k+\{a_n\}T \qquad (4)$$

Referring to Fig.7,{ X1l } is obtained by taking account of the equilibrium at pin support.

$$\{X_{1l}\}=\{e\}T+\{f_0\}H+\{f_1\}V$$

$$\{f_0\}=\begin{Bmatrix} \sin(\theta-\phi_0)/\sin2\theta \\ 0 \\ \sin(\theta+\phi_0)/\sin2\theta \\ 0 \end{Bmatrix}$$

$$\{f_1\}=\begin{Bmatrix} \cos(\theta-\phi_0)/\sin2\theta \\ 0 \\ -\cos(\theta+\phi_0)/\sin2\theta \\ 0 \end{Bmatrix}, \quad \{e\}=\begin{Bmatrix} 1/(1+k) \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

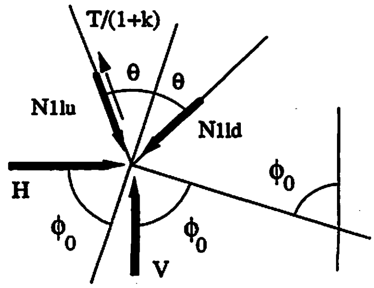

Fig.7. Equilibrium at pin supported

Where V is the vertical reaction at the pin support and it is explicitly expressed with the geometrical parameter and external forces.H indicates the unknown horizontal reaction at the pin support.Unknown T and H will be decided so that Qnru and Qnrd take zero in Eq.(4). After substituting H and T into Eqs.(1) and (2), all forces are obtained finally. It is confirmed that Nnru and Nnrd satisfy the equilibrium at the center hinged point. After all, the internal forces of CSA can be decided by the only equilibrium conditions. The calculations of point displacements are based upon the principle of virtual work.

3.2. Winch tension force

Fig.8 and 9 show experimental relations between the winch tension force: T and the central height:h in case of the model without membrane,with membrane respectively. The total weight of two models are 180kg,190kg respectively. Experimental four kinds of T and theoretical two kinds of T are defined as follows. Tui means the instantaneous winch tension just after stopping winding up at a height. Tus is called the stable winch force after a perturbation during winding up. Tdi,Tds mean the instantaneous,the stable winch tension respectively in case of winding back process. Tc means two times of the thrust reaction in arch statics under uniformly lateral load. Tua/k=0 means two times of the numerical cable tension without friction based on the above-mentioned analysis,and Tua/k=x means two times of the cable tension in case of the model which friction coefficient: k is x. As shown in these Figs, 1)Tua/k=0 exceeds Tc in the range of less than 425cm rise, 2)Tus, Tui is close to Tua/k=0, Tua/k=0.03 respectively, 3) Tdi is almost same as Tds, but Tui is different from Tus . As shown in Fig.9, beyond 500 cm rise in case of the membrane model, Tui and Tus change to increase because the membrane begins to works in tension.

4. Conclusion

Trial construction test on a wooden cylindrical type of CSA with 11m span,was sucssecefully carried out based on the simple idea.On the other side,the structural analysis was developed by treating the mechanical model of the pulley-joints as 'Pin joint with Friction between Pulley and Cable'. The experimental winch tension forces were examined theoretically.

5. Acknowledgement

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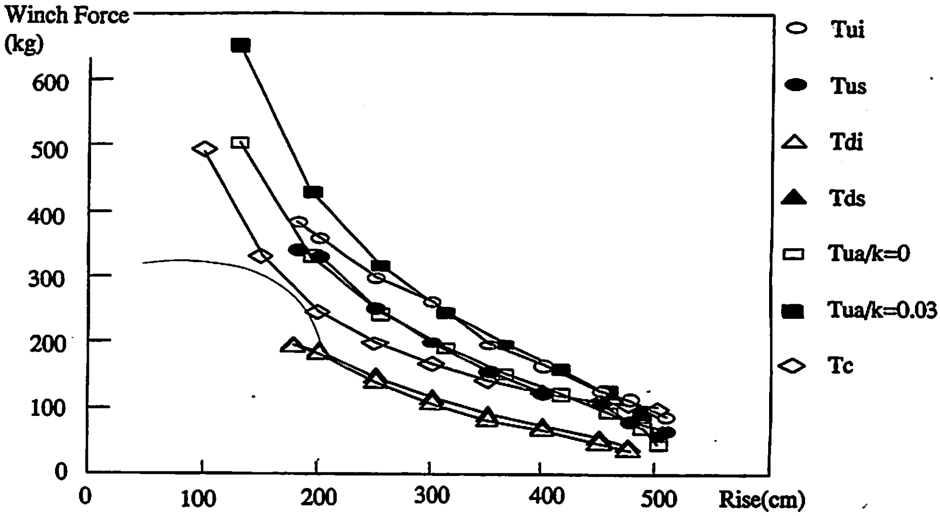

Fig.8. Rise - Winch force (no membrane model)

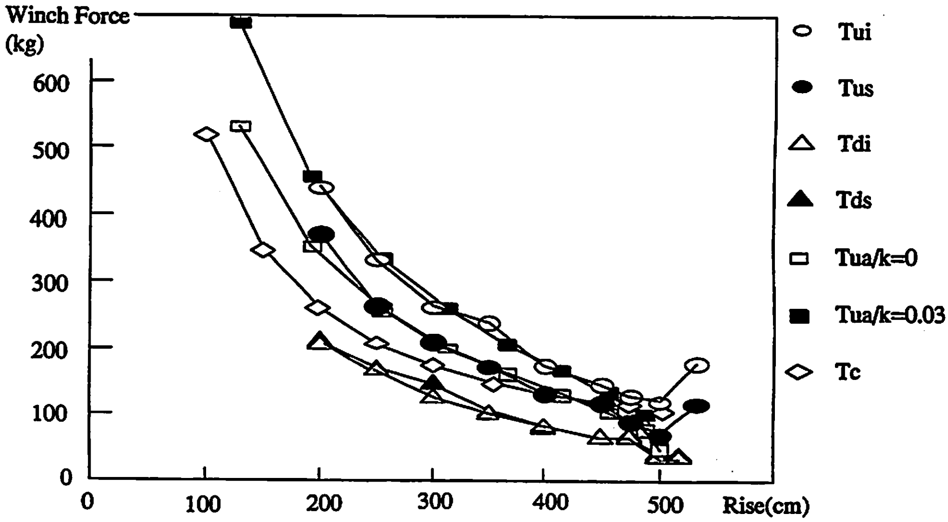

Fig.9 Rise - Winch force (membrane model)

6. References

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