SCISSORS ARCH WITH ZIGZAG-CABLE THROUGH PULLEY-JOINTS

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ABSTRACT
CSA(Cable Scissors Arch) has a possibility of introducing a dynamical expression into the architectural space design,because of the continuously changing form structure.The structural analysis of CSA is developed here,by a mathematical treatment of the pulley-joints as 'Slipping Pin Joints without Friction between cable and pulley'. According to the computational results of CSA compared with SA(Scissors Arch) and STA(Scissors Trussed Arch),it is shown numerically that the zigzag cable plays an important roles for CSA in the point of structural efficiency improvement.

1.INTRODUCTION

The continuously changing form structure has a new possibility to introduce a dynamical expression into the architectural space design. Scissors-type may be considered as one of the possible structures to change the form continuously. In case of the realization of scissors-type expandable structures,one of the most important problems is concerned with the rational method of expanding and rocking all the time. The author proposed a new type of expandable arch 'CSA(Cable Scissors Arch) shown in Fig.1,2' which finds out a solution to these problems at the same time (Kokawa,1995). As shown in Fig.2,a zigzag flexible cable passes through the pulleys which are installed at the connection points between the scissors units. As a tension force always works in the cable under the gravity load,CSA will be a stable and statically determinate structure all the time. Although it doesn't have the chords,CSA might be considered as a kind of the truss structure which resists mainly axial forces.It is supposed intuitively that CSA has far the better mechanical efficiency in the points of strength and rigidity,compared with scissors structures without cable 'SA(Scissors Arch) as shown in Fig.3'. However the evaluations have not been clear numerically yet.
The main purpose of this paper is to investigate numerically the structural behavior of CSA. First of all the statical analysis of CSA is developed here,by a mathematical treatment of the pulley-joint as 'Slipping Pin Joint without Friction between cable and pulley'. And then,the structural behavior of CSA is numerically discussed by comparing with SA and 'STA(Scissors Trussed Arch which consists of scissors units and pin-jointed strut member shown in Fig.4)'.

2.ANALYTICAL METHOD OF CSA

2.1 Cable Arrangements and Mathematical Treatment of Pulley Joint

Fig.5 shows the cable arrangements of CSA which is the subject of this paper ,although

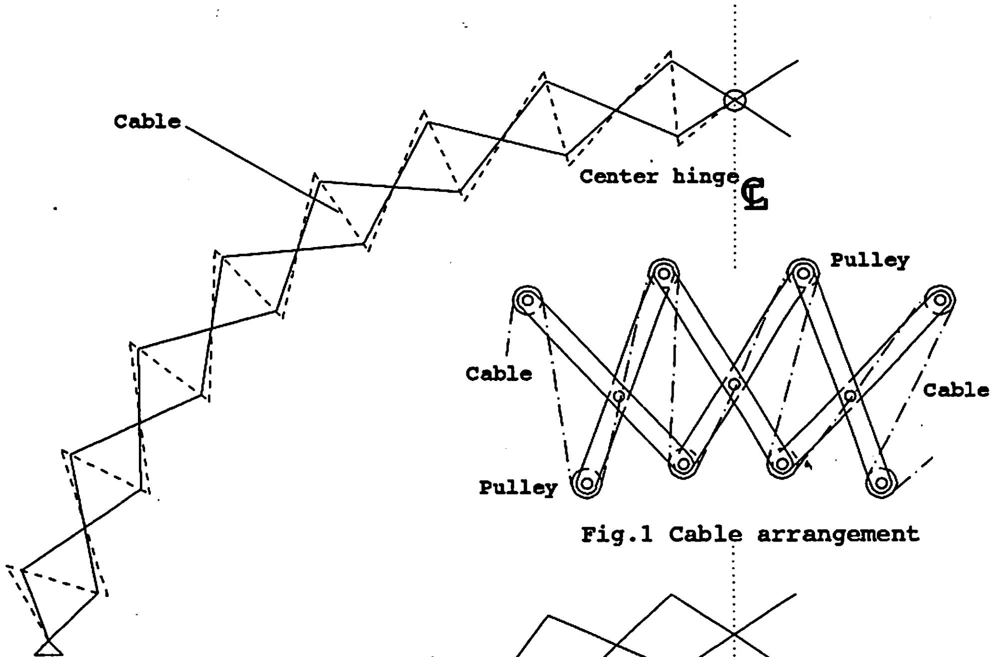


Fig.1 Cable arrangement

Fig.2 CSA (Cable Scissors Arch)

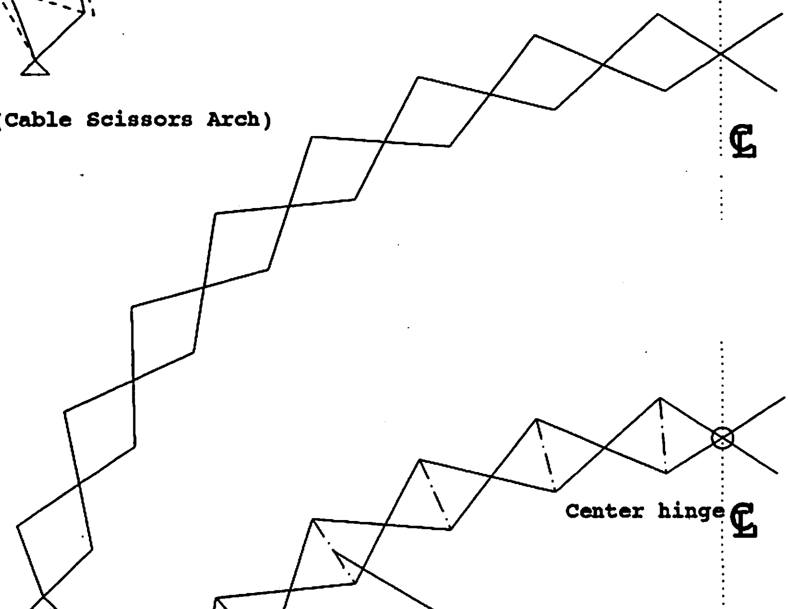


Fig.3 SA (Scissors Arch)

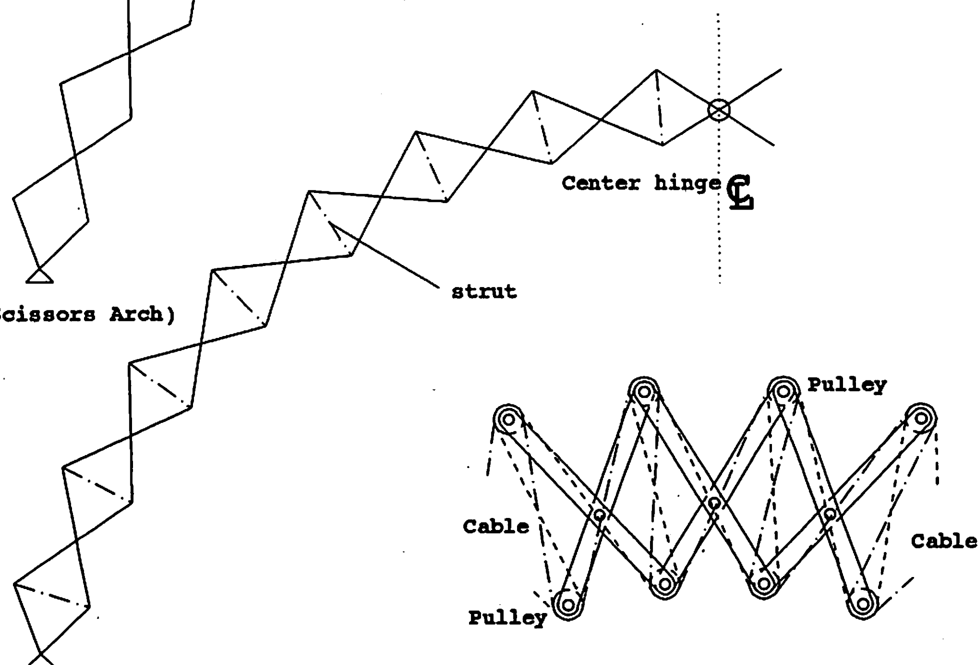


Fig.5 Cable arrangement(double)

Fig.4 STA (Scissors Trussed Arch)

a bit different arrangements from Fig.1. Each cable starts at the central hinged point and zigzag passes through pulleys to each supporting end, and then return back to the center by the same zigzag passing through.Although small friction forces actually exist between the cable and the pulley joint,the mathematical model of the joint is treated as 'Slipping Pin Joint without Friction between cable and pulley'. Therefore,it is supposed that each cable has a constant tension force whole the structure.

2.2 Expressions of Transfer Matrix

Fig.6 shows all kinds of forces in unit[i] and [i+1]. Considering the euilibrium($\Sigma X=0$, $\Sigma Y=0$) at joint no.① and ②, the relation between the right side forces vector:$\{X_{ir}\}$ and the left side vector:$\{X_{il}\}$,is given in Eq.(1).

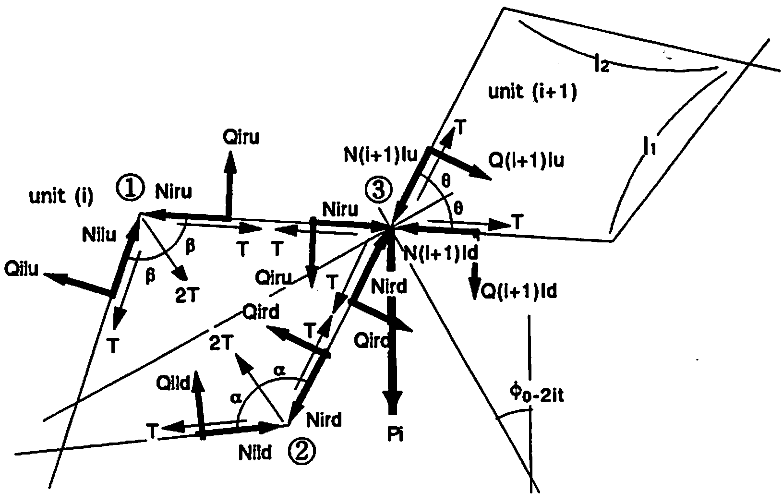


Fig.6 Internal and external forces

$$\{X_{ir}\}=[A]\{X_{il}\}+\{a\}T \qquad \text{-----(1)}$$

Where T denotes the unknown cable tension force which is constant whole the structure,and

$$\{X_{ir}\}=\begin{Bmatrix} N_{iru} \\ Q_{iru} \\ N_{ird} \\ Q_{ird} \end{Bmatrix}, \quad \{X_{il}\}=\begin{Bmatrix} N_{ilu} \\ Q_{ilu} \\ N_{ild} \\ Q_{ild} \end{Bmatrix},$$

$$\{a\}=\begin{Bmatrix} 2(1+\cos\beta)\cos\beta \\ 2(1+\cos\beta)\sin\beta \\ 2(1+\cos\alpha)\cos\alpha \\ -2(1+\cos\alpha)\sin\alpha \end{Bmatrix}, \quad [A]=\begin{bmatrix} -\cos 2\beta & -\sin 2\beta & 0 & 0 \\ -\sin 2\beta & \cos 2\beta & 0 & 0 \\ 0 & 0 & -\cos 2\alpha & \sin 2\alpha \\ 0 & 0 & \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

And,the relation between $\{X_{ir}\}$ and $\{X_{(i+1)l}\}$ is shown in Eq.(2), considering the equilibrium of point no.③.

$$\{X_{(i+1)l}\}=[B]\{X_{ir}\}+\{b_i\}P_i \qquad \text{-----(2)}$$

$$\{X_{(i+1)l}\}=\begin{Bmatrix} N_{(i+1)lu} \\ Q_{(i+1)lu} \\ N_{(i+1)ld} \\ Q_{(i+1)ld} \end{Bmatrix}, \quad \{b_i\}=\begin{Bmatrix} -\dfrac{\cos(\varphi_0 - 2it - \theta)}{\sin 2\theta} \\ 0 \\ \dfrac{\cos(\varphi_0 - 2it + \theta)}{\sin 2\theta} \\ 0 \end{Bmatrix}$$

$$[B]=\begin{bmatrix} 0 & -(1+\xi)\operatorname{cosec} 2\theta & 1 & -\left(1+\frac{1}{\xi}\right)\cot 2\theta \\ 0 & 0 & 0 & \frac{1}{\xi} \\ 1 & (1+\xi)\cot 2\theta & 0 & \left(1+\frac{1}{\xi}\right)\operatorname{cosec} 2\theta \\ 0 & \xi & 0 & 0 \end{bmatrix}, \quad \xi=\frac{l_2}{l_1}$$

Eq.(3) which indicates the relation between $\{X_{il}\}$ and $\{X_{(i+1)l}\}$,is derived from Eqs.(1) and (2).

$$\{X_{(i+1)l}\}=[C]\{X_{il}\}+\{d\}T+\{b_i\}P_i \qquad \text{-----(3)}$$

Where, [C]=[B][A]

$$=\begin{bmatrix} (1+\xi)\dfrac{\sin 2\beta}{\sin 2\theta} & -(1+\xi)\dfrac{\cos 2\beta}{\sin 2\theta} & -\cos 2\alpha-\left(1+\frac{1}{\xi}\right)\cot 2\theta \sin 2\alpha & \sin 2\alpha-\left(1+\frac{1}{\xi}\right)\cot 2\theta \cos 2\alpha \\ 0 & 0 & \frac{1}{\xi}\sin 2\alpha & \frac{1}{\xi}\cos 2\alpha \\ -\cos 2\beta-(1+\xi)\cot 2\theta \sin 2\beta & -\sin 2\beta+(1+\xi)\cot 2\theta \cos 2\beta & \left(1+\frac{1}{\xi}\right)\dfrac{\sin 2\alpha}{\sin 2\theta} & \left(1+\frac{1}{\xi}\right)\dfrac{\cos 2\alpha}{\sin 2\theta} \\ -\xi \sin 2\beta & \xi \cos 2\beta & 0 & 0 \end{bmatrix}$$

$$\{d\}=[B]\{a\}=\begin{Bmatrix} -(1+\xi)\dfrac{1}{\sin 2\theta}2(1+\cos\beta)\sin\beta+2(1+\cos\alpha)\cos\alpha+\left(1+\frac{1}{\xi}\right)\cot 2\theta\, 2(1+\cos\alpha)\sin\alpha \\ -\frac{1}{\xi}2(1+\cos\alpha)\sin\alpha \\ 2(1+\cos\beta)\cos\beta+(1+\xi)\cot 2\theta \cdot 2(1+\cos\beta)\sin\beta-\left(1+\frac{1}{\xi}\right)\dfrac{1}{\sin 2\theta}2(1+\cos\alpha)\sin\alpha \\ \xi\, 2(1+\cos\beta)\sin\beta \end{Bmatrix}$$

From Eq.(3),$\{X_{il}\}$ can be expressed by using $\{X_{1l}\}$ which indicates the end force vector connected to the pin support,as written in Eq.(4).

$$\{X_{il}\}=[C]^{(i-1)}\{X_{1l}\}+\left(\sum_{k=1}^{i-1}[C]^{(i-k-1)}\right)\{d\}T+\sum_{k=1}^{i-1}[C]^{(i-k-1)}\{b_k\}P_k \qquad \text{-----(4)}$$

Therefore,$\{X_{nl}\}$ is obtaned by substituting i=n into Eq.(4).

$$\{X_{nl}\}=[C]^{(n-1)}\{X_{1l}\}+\left(\sum_{k=1}^{n-1}[C]^{(n-k-1)}\{d\}T+\sum_{k=1}^{n-1}[C]^{(n-k-1)}\{b_k\}P_k\right)$$

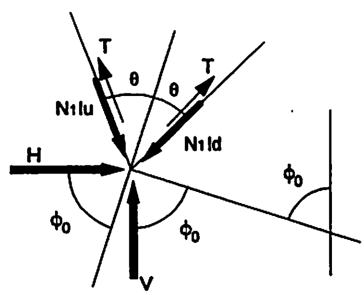
Finally,the other end force vector: $\{X_{nr}\}$ which is connected to the central hinged point, is expressed in Eq.(5).

$$\{X_{nr}\}=[A][C]^{(n-1)}\{X_{1l}\}+[A]\left(\sum_{k=1}^{n-1}[C]^{(n-k-1)}\right)\{d\}T+[A]\sum_{k=1}^{n-1}[C]^{(n-k-1)}\{b_k\}P_k \qquad \text{-----(5)}$$

2.3 Expression of $\{X_{1l}\}$ and $\{X_{nr}\}$

Refering to Fig.7 , $\{X_{1l}\}$ is obtained by taking account of the equilibrium at the pin support.

$$\{X_{1l}\}=\{e\}T+\{f\} \quad , \quad \{e\}=\begin{Bmatrix}1\\0\\1\\0\end{Bmatrix} \quad , \quad \{f\}=\begin{Bmatrix} H\dfrac{\sin(\theta-\varphi_0)}{\sin 2\theta}+V\dfrac{\cos(\theta-\varphi_0)}{\sin 2\theta} \\ 0 \\ H\dfrac{\sin(\theta+\varphi_0)}{\sin 2\theta}-V\dfrac{\cos(\theta+\varphi_0)}{\sin 2\theta} \\ 0 \end{Bmatrix} \qquad \text{-----(6)}$$



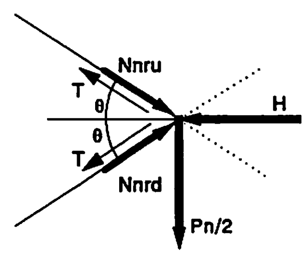


Fig.7 Equilibrium at supporting end　　Fig.8 Equilibrium at center hinge

Where H and V indicate the horizontal and vertical reaction at the pin support respectively,and they are explicitly expressed with the geometrical parameter and external forces because CSA is a kind of 3-hinged arch which means a statically determinate structure.

On the other side,Refering to Fig.8 , $\{\overline{X}_{nr}\}$ is obtained by the equilibrium at the center hinged point.

$$\{\overline{X}_{nr}\}=\{e\}T+\{g\} \quad , \quad \{g\}=\begin{Bmatrix} \dfrac{H}{2\cos\theta}-\dfrac{P_n}{4\sin\theta} \\ 0 \\ \dfrac{H}{2\cos\theta}+\dfrac{P_n}{4\sin\theta} \\ 0 \end{Bmatrix} \qquad \text{-----(7)}$$

And then, the following equation(8) concerning with T is derived from $\{X_{nr}\}=\{\overline{X}_{nr}\}$.

$$\left([A][C]^{(n-1)}\{e\}+[A]\left(\sum_{k=1}^{n-1}[C]^{(n-k-1)}\right)\{d\}+\{a\}-\{e\}\right)T$$
$$=\{g\}-[A][C]^{(n-1)}\{f\}-[A]\sum_{k=1}^{n-1}[C]^{(n-k-1)}\{b_k\}P_k \qquad \text{-----(8)}$$

Four values of T are obtained from Eq.(8),but they are perfectly same.After substituting T into Eqs.(4) and (1),all forces are obtained finally. After all,the only equilibrium conditions are needed to get the internal forces of CSA.The calculations of point displacements are based upon the principle of virtual work.

3.ANALYSIS OF SA

Because SA is a kind of 2-hinged arch as shown in Fig.3,the vertical reaction:V is obtained easily. It is possible to analyse here by treating H as the unknown horizontal reaction at the supporting point of SA. First af all, as the zigzag cable does not exist in this case,T has to take zero at all equations of the previous **section 2**. And $\{X_{11}\}$ is shown as follows.

$$\{X_{11}\}=\{\zeta\}V+\{\eta\}H \quad , \quad \{\zeta\}=\begin{Bmatrix} \dfrac{\cos(\theta-\varphi_0)}{\sin 2\theta} \\ 0 \\ -\dfrac{\cos(\theta+\varphi_0)}{\sin 2\theta} \\ 0 \end{Bmatrix} \quad , \quad \{\eta\}=\begin{Bmatrix} \dfrac{\sin(\theta-\varphi_0)}{\sin 2\theta} \\ 0 \\ \dfrac{\sin(\theta+\varphi_0)}{\sin 2\theta} \\ 0 \end{Bmatrix} \qquad \text{-----(9)}$$

And then,taking consideration of the symmetrical conditions concerning with the central line from $\{X_{nr}\}=\{X_{(n+1)l}\}$,the following equation about H is obtained.

$$\left([C]^n-[A][C]^{(n-1)}\right)\{\eta\}H$$
$$=[A]\sum_{k=1}^{n-1}[C]^{(n-k-1)}\{b_k\}P_k-\sum_{k=1}^{n-1}[C]^{(n-k)}\{b_k\}P_k+\left([A][C]^{(n-1)}-[C]^n\right)\{\zeta\}V \qquad \text{-----(10)}$$

Four values of H are obtained from Eq.(10),but they are perfectly same.
The analytical method of STA was described at the previous paper(Kokawa,1995).

4.NUMERICAL RESULT AND DISCUSSIONS

The structural behaviour of CSA is investigated numerically in comparison with that of SA and STA as for the model with following specific parameters.
General form: Circular arch,10m span and 4m rise.
Member of scissors unit: Sectional area =100cm^2, inertia moment=833cm^4 ,Young's modulus=70t/cm^2, member length=103.71cm (l_1=49.00cm , l_2=54.71cm) and number of units=8(one side).
Cable: Sectional area=1cm^2,Young's modulus=2000t/cm^2.
In case of STA,the sectional area of the pin-jointed strut member becomes 2cm^2, because of the same sectional area as CSA's double cables.
Loading: Vertically concentrated 10kg every cross point
The vertical displacement ,axial force and shear force of CSA,SA and STA is shown in Fig.9,10 and 11,respectively.

4.1 CSA and SA

Refering to Fig.9 and 10, it is recognized easily that the zigzag cable produces the improvement of mechanical efficiency in the points of strength and rigidity. As CSA and SA are both statically determinate structures,a first local failure leads immediatelly to the general failure of these structure.And the bending moment at the cross point of scissors unit,that is, the shear force is seemed to be deeply related to the failure. The ratio of the maximum shear forces between SA and CSA ,SA/CSA becomes 258.6Kg/44.1Kg ,so the strength of CSA is about 6 times of SA. On the other side,the distributions of vertical displace-ments are similar to each other,but the magnitude is quite different.For example, the maximum displace-ments of both structures take place in the central point,but the ratio of them,CSA/SA becomes 128.69 mm / 4.035mm(=31.9). Therefore it is recognized that the zigzag cable is very useful for the improvement of structural rigidity.

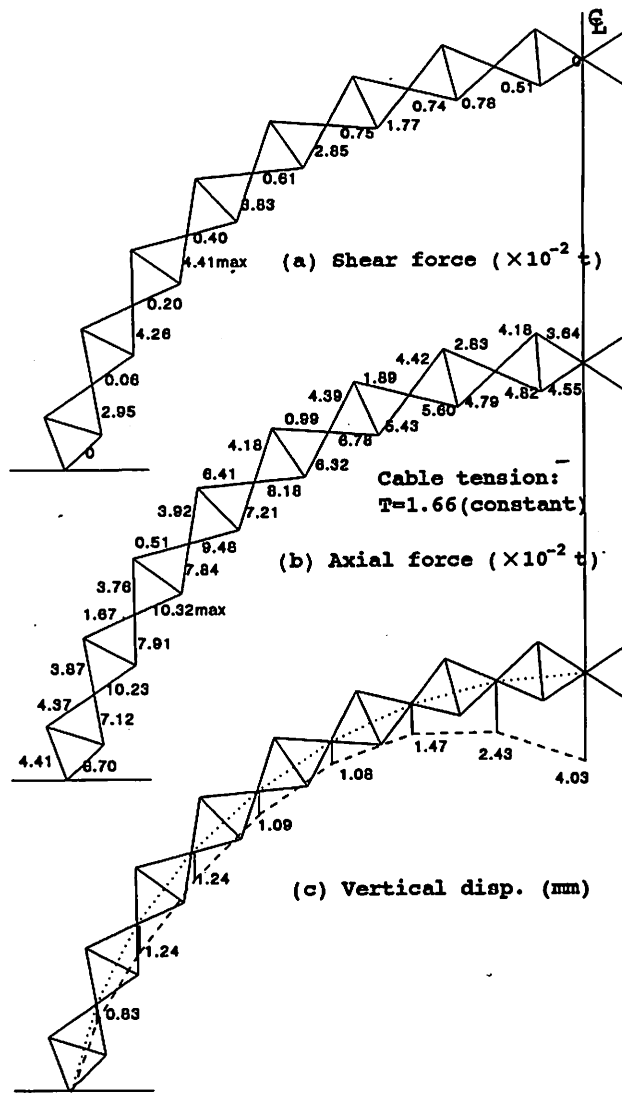


Fig.9 CSA (Cable Scissors Arch)

4.2 CSA and STA

Refering to Fig.9 and 11,STA is superior to CSA in the structural efficiency,but the diference is not so big as between CSA and SA. The ratio of maximum shear forces CSA / STA(=44.1Kg / 20.3Kg) becomes 2.16,so it is estimated that the strength of STA is about 2 times of CSA under such a loading.Furthermore,two bending moments at a cross point every scissors unit,are almost same in case of STA,but the one is very small and the other is concentrated in case of CSA. Distributions of axial forces have same tendency,and the ratio of the maximum values CSA/STA(=103.2Kg/77.6Kg) becomes 1.33. T(tension force of CSA's cable) is constant 16.6 Kg,so 33.2Kg in double cables. This value is almost same as the average tension force of the pin-jointed strut in STA. The maximum vertical displacement of the both models take place in the center points,and the ratio CSA/STA(=4.035mm/3.843mm) becomes 1.05,although the mode of displacements is different from each other.

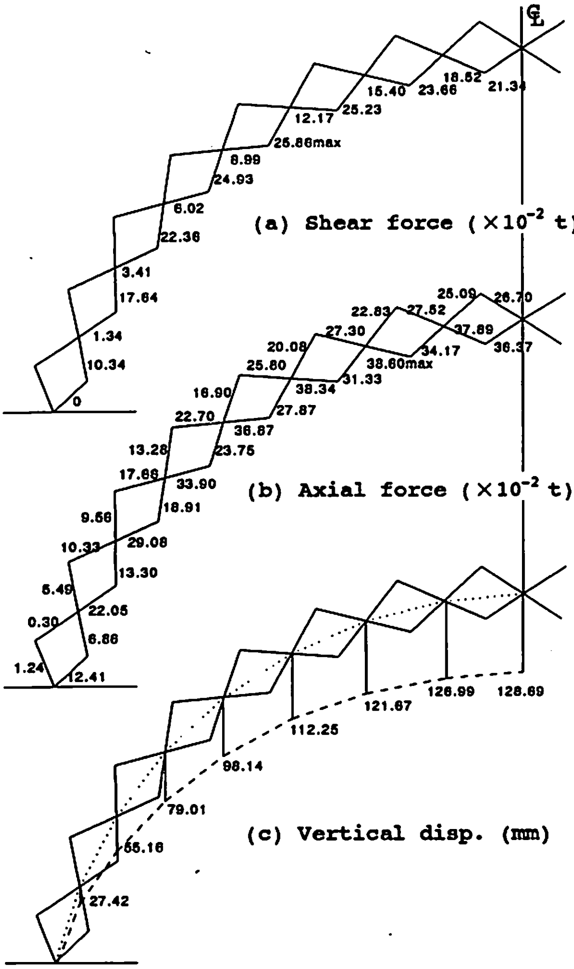


Fig.10 SA (Scissors Arch)

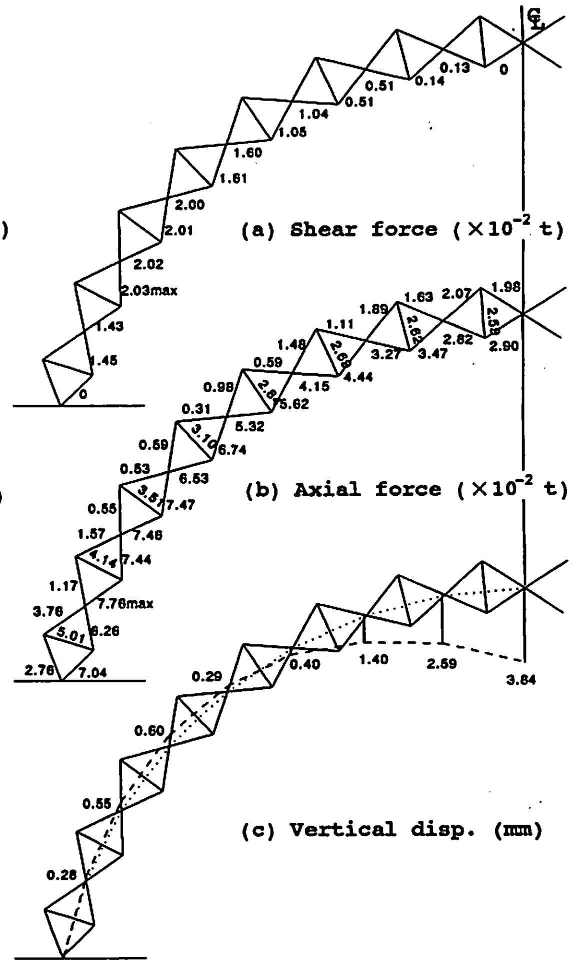


Fig.11 STA (Scissors Trussed Arch)

5.CONCLUSION

Treating the mechanical model of the pulley-joint as 'pin joint without friction beween pulley and cable',the structural analysis of CSA has been developed. The structural behaviour of CSA was compared numerically with SA and STA as for a specific geometrical model. As the results,it is confirmed numerically that the zigzag cable plays an important roles for CSA in the point of structural efficiency improvement.

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Reference

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