A TRIAL OF EXPANDABLE ARCH

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ABSTRACT

In case of the realization of scissors–type expandable structures,one of the most important problems is concerned with the rational method of expanding and rocking.

This paper describes a new type of expandable structure which consists of three–hinged arch scissors and zigzag flexible cables through pulleys with them. The changing form structure is controlled only by winding up or back tools like a winch,and a special stable truss arch without chords is always gained during the operation because the cable is effective against tension force under gravity load.

The contents of this paper are (1) Basic idea of consrtuction, (2) Trial construction of span 5–m a)steel arch and b)wooden dome,and (3) Analytical model for the structural statics.

1.INTRODUCTION

The changing form structure proposed in this paper is concerned with an arch type,and it basically comes from scissors mechanism. A flexible frame for drying clothes as well as an expandable gate,are well known examples of simple scissors structures in our living equipment. The expandable wall in Mongolian traditional houses 'Yurt' is called 'Khana' (Faegre,1979) which has same scissors structure like as 'Grid shell'(Otto,1974). It is said that the modern study on scissors–type changing form structures was initiated by Pinero who proposed a transportable packed auditorium(Pinero,1962). Studies on the structural system(Escrig,1985),the geometrical compatibility conditions(Puertas Del Rio,1991),structural analysis(Escrig and Valcarcel,1986) and the ideas for the stabilization(Rosenfeld and Logcher,1988) have been developed since that. However,scissors–type changing form structures have not been so popular until today. It might be the reason why there are no good ideas about (1) a rational method of expanding and rocking(Kwan and others,1993) and (2) an improvement of the low efficiency caused by the bending system in only scissors type.Thereupon,the author proposed a new type of expandable arch scissors structure(Kokawa and Watanabe,1994a) which finds out a solution to these problems at the same time.

2.BASIC IDEA

As shown in Fig.1,two linking scissors arch structures are consisted of scissors units(Fig.2), and settled on a near ground. And then, a zigzag flexible cable passes through the pulleys which are installed at the connection points between the scissors units,as shown in Fig.3. During winding up the cable by a winch,the structure is going to expand and be forced to lift up as a result,as shown in Fig.1. On the other side,it will be shortened and go down by its self−weight during the winding back. A big cable tension force may be needed in order to lift up and down the structure which is in a near ground. The solution of this problem may be found out by the ideas that a pneumatic membrane is used as not only a roofing material but also a supplemental instrument for lifting up and down at the lowest location. The weight of the structure will be in equilibrium with a compression in the strut and a tension in the cable, through the whole operation. As a tension force always works in the cable owing to the existense of gravity,this cable scissors structure will be stable all the time. It might be considered as a kind of truss which resists mainly axial forces,although it doesn't have the chords. Therefore,this structure has far the better mechanical efficiency in the points of strength and

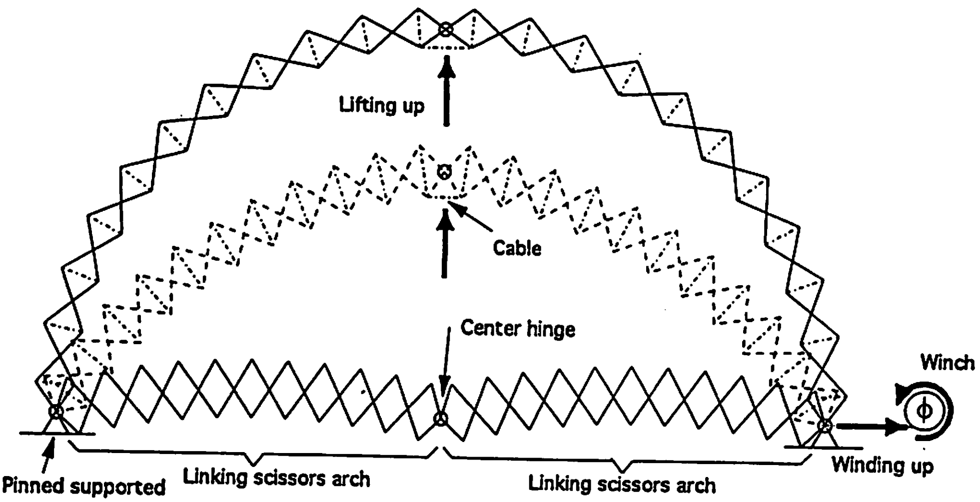


Fig.1 Basic idea

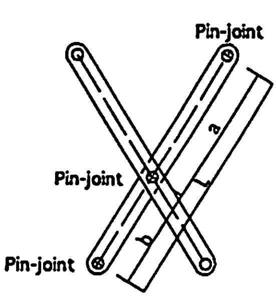


Fig.2 Scissors unit

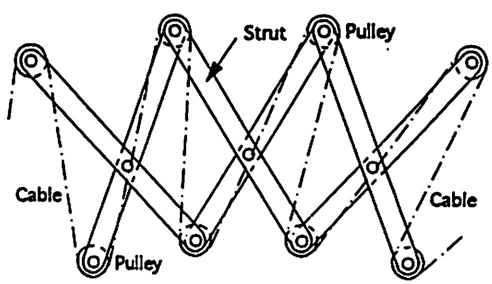


Fig.3 Cable arrangement

rigidity,compared with scissors structures without cable. Although the length ratio of a scissors unit,a/b,is considered to be constant in the whole structure here,it is not so difficult to make a freely curved arch−like structure by changing a/b every unit.

After all,it is possible to give a continuous change of arch−type scissors structures which are always stable statistically,by only winding up and back a cable. Furthermore,it is possible to construct the arch and dome with a designed form by only providing a cable tension,after setting it at a near ground. As this method is easy,simple and safe,its construction period will be drastically short.

3.CONSTRUCTION TEST

3.1 Steel Arch Model

Fig.4 shows two stages of situations during lifting up a span 5.3m arch−type cable scissors. Fig.5 shows the diagram about this test. This structure uses six different type of members A,B,C,D,E,F which respective numbers are 20,20,2,2,2,2 . These members are made from channel steel([−200·50·4.5,[−250·50·4.5),flat bar(F.B.−38·4.5),and Fig.6 shows the detail of member A and B as examples. As every unit has same length a(=235 mm) and b(=213 mm) in this model,so each linking scissors structure becomes a partial circular arch.The both ends of this structure are pinned supported by a steel fitting on a circular R.C. foundation ring with 5.3 m base diameter,and the center point is a hinged connection to member E,F. Zigzag flexible cables with nomial diameter ϕ 2,are passed through pulleys which are installed at the both side connection points between scissors units,as shown in Fig.7. First of all,an investigation on the unstable order of the structure without cables is as follows.

n=4, s=88, r=40, k=67, where n, s, r and k are respective number of reaction elements, members, rigidly connected members and joints. Substituting these into its discriminant,

$m=n+s+r-2k=4+88+40-2\cdot 67=-2<0$

This equation means the structure without cables has a second instability order, and it will be possible to change a form of the structure with rigid body mode from the earlier state without an arbitary strain energy. Corresponding to the movement of one point,the

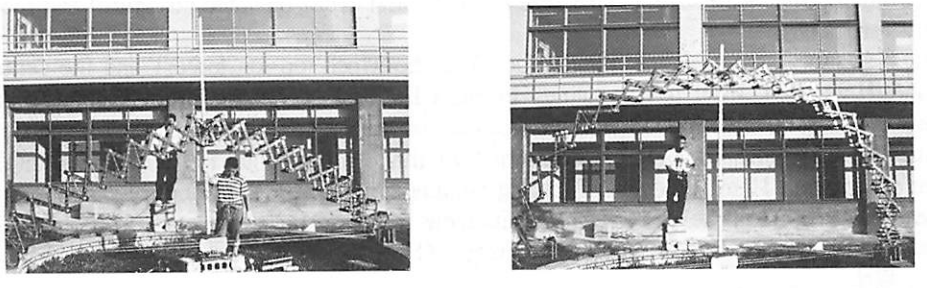


Fig.4 Lifting up

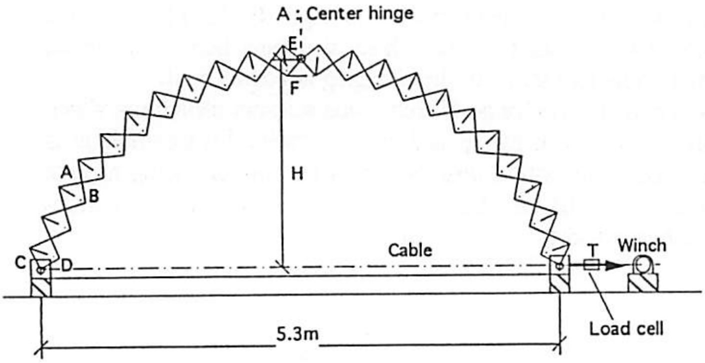


Fig.5 Diagram of construction test

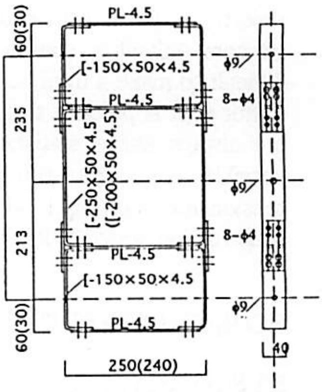


Fig.6 Member A(B)

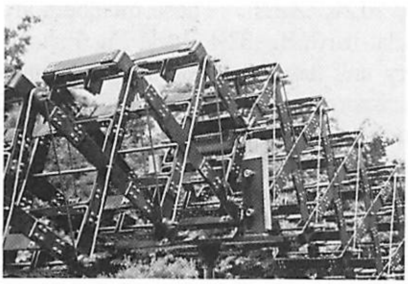


Fig.7 Pulley and zigzag cable

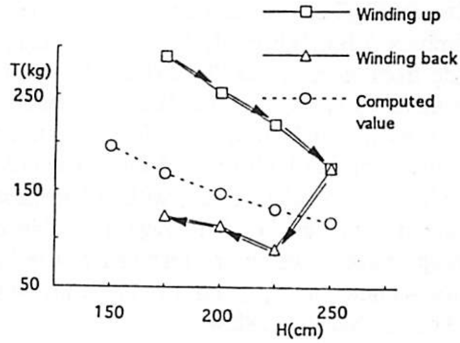


Fig.8 T-H (steel arch)

other points will be fixed uniquely. In order to keep the symmetry of the unstable structure as long as possible during the movement,the centre points have to move almost vertically. Thereupon,each cable starts at the centre point and zigzag passes through pulleys to each end,and then connects to a winch that is installed on another R.C. foundation ring,as shown in Fi.5. Owing to this method,as each length variation of cable during winding up and back is almost same,the centre point will be constrained horizontally (that is to say,this structure becomes first order unstable structure) and move only vertically.

Fig.8 shows an experimental relation between the winch tension force:T and the central height:H,where T and H are explaine in Fig.5. In this test,H varies from 150 cm to 250 cm and a lateral deformation occured because of the low horizontal bending rigidity at small H. So,a kind of stay was connected to the central point. From this result,(1)T decreases,when H increases, (2)T during winding up,exceeds T during winding back at the same H. (1) can be understood qualitatively from the thrust reaction in arch statics under uniform load as shown following equation (1).

$$R_S = \frac{W_T L}{8H} \qquad \cdots\cdots (1)$$

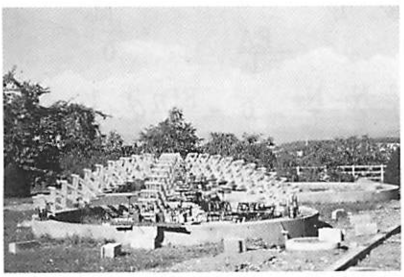
Where R_S, W_T and L is thrust force,total weight of the structure and span. In this test,2 times R_S corresponds to a theoretical winch tension:T_S,and substituting W_T=222.3 kg,L=530 cm into eq.(1),

$$T_S = \frac{29454}{H} \quad \cdots\cdots (2)$$

Substituting H=175cm,200cm and 225cm into eq.(2),T_S becomes 168kg,147kg and 131kg respectively. These T_S are located between the both experimental winch forces at the winding up and back,and all T_E/T_S are in the range from 1.18 to 1.24,where T_E means the average value at the winding up and back. T_E might be considered as the winch force if there is no friction in the pulley.

3.2 Wooden Dome Model

A wooden cable scissors dome structure consists of same six wooden cable scissors arches arranged in a radial manner from the centre. Each arch unit is almost of same type as the steel one predescribed at 3.1. An experimental result concerned with the relation between winch tension force :T and rise in dome:H,is shown in Fig.9. Total weight of this dome:W_D = 110 kg,therefore theoretical total winch tension force:T_D becomes 14575/H which is between the both experimental winch forces. Fig.10 shows three stages during lifting up and down of the dome with base diameter 5.3 m.



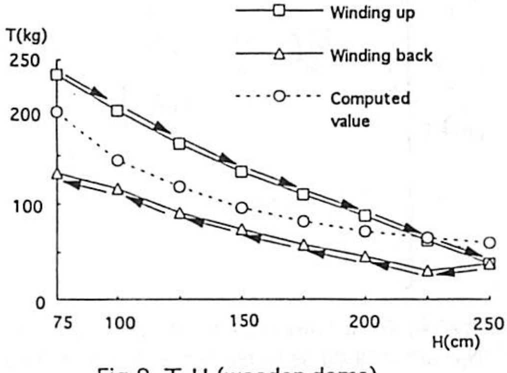


Fig.9 T-H (wooden dome)

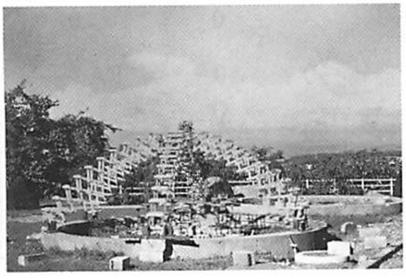


Fig.10 Lifting up (wooden dome)

4. METHODS OF STRUCTURAL ANALYSIS

4.1 Frame Model

Fig.11 shows a typical strut of scissors unit drawn with the real line. Forces F_X and F_Y, as well as corresponding displacements U and V,exist at nodes i,j and k.They have positive directions as shown in the figure,and local coordinates are used. The strut element is assumed to have a compressive force and a uniform flexural stiffness EI and axial stiffness EA over its length. The derivation of stiffness matrix for the strut is based upon the the exact solution of buckling equation for the beam element concerned with bending and the approximate expression for the truss element concerned with compression,as described at the previous paper(Kokawa and Watanabe,1994b). The stiffness equation is,

$$\{F\}=[K]\{\Delta\} \qquad (3)$$

Where $\{F\}^T=(F_{Xi},F_{Yi},F_{Xj},F_{Yj},F_{Xk},F_{Yk})$: nodal forces vector, $\{\Delta\}^T=(U_i,V_i,U_j,V_j,U_k,V_k)$: nodal displacements vector, [K] : stiffness matrix, $[K]=[K_L]+[K_N]$, $[K_L]$: Linear term, $[K_N]$: nonlinear term, [K], $[K_L]$, $[K_N]$: 6×6 symmetry matrix, $[K_L]$,$[K_N]$ is as follows, respectively.

$$[K_L]=\begin{bmatrix} \frac{EA}{l_{ij}} & 0 & -\frac{EA}{l_{ij}} & 0 & 0 & 0 \\ & \frac{EI\eta}{l_{ij}^2}-\frac{N_{ij}}{l_{ij}} & 0 & -\frac{EI\eta}{l_{ij}}\left(\frac{1}{l_{ij}}+\frac{1}{l_{jk}}\right)+\frac{N_{ij}}{l_{ij}} & 0 & \frac{EI\eta}{l_{ij}l_{jk}} \\ & & EA\left(\frac{1}{l_{ij}}+\frac{1}{l_{jk}}\right) & 0 & -\frac{EA}{l_{jk}} & 0 \\ & & & EI\eta\left(\frac{1}{l_{ij}}+\frac{1}{l_{jk}}\right)^2-\frac{N_{ij}}{l_{ij}}-\frac{N_{jk}}{l_{jk}} & 0 & -\frac{EI\eta}{l_{jk}}\left(\frac{1}{l_{ij}}+\frac{1}{l_{jk}}\right)+\frac{N_{jk}}{l_{jk}} \\ & \text{SYM.} & & & \frac{EA}{l_{jk}} & 0 \\ & & & & & \frac{EI\eta}{l_{jk}^2}-\frac{N_{jk}}{l_{jk}} \end{bmatrix}$$

where N_{ij}, N_{jk} a compression force in member of i−j,j−k,respectively and η (function of N_{ij},N_{jk}) are treated as linear terms in this numerical analysis.

$$[K_N]=\begin{bmatrix} 0 & -EA\frac{V_i-V_j}{2l_{ij}^2} & 0 & EA\frac{V_i-V_j}{2l_{ij}^2} & 0 & 0 \\ & EA\frac{U_i-U_j}{2l_{ij}^2} & EA\frac{V_i-V_j}{2l_{ij}^2} & -EA\frac{U_i-U_j}{2l_{ij}^2} & 0 & 0 \\ & & 0 & -EA\left(\frac{V_i-V_j}{2l_{ij}^2}+\frac{V_j-V_k}{2l_{jk}^2}\right) & 0 & EA\frac{V_j-V_k}{2l_{jk}^2} \\ & & & EA\left(\frac{U_i-U_j}{2l_{ij}^2}+\frac{U_j-U_k}{2l_{jk}^2}\right) & EA\frac{V_j-V_k}{2l_{jk}^2} & -EA\frac{U_j-U_k}{2l_{jk}^2} \\ & \text{SYM.} & & & 0 & -EA\frac{V_j-V_k}{2l_{jk}^2} \\ & & & & & EA\frac{U_j-U_k}{2l_{jk}^2} \end{bmatrix}$$

On the other side,the ordinary linear equation for the pin−jointed truss member is used here as the stiffness equation for the tension cable. Each stiffness matrix in local coordinate is transformed into an overall reference coordinate by employing each transformation matrix,and then superimposed to yield the total structure stiffness matrix. Newton−Raphson technique is adopted basically in order to get the numerical solution of the total stiffness equations.

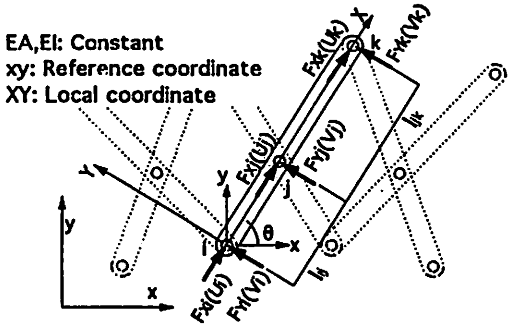


Fig.11 Nodal forces and disp. of a strut

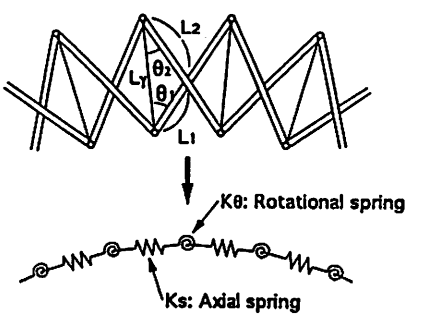


Fig.12 Frame model -> Spring model

4.2 Spring Model

In order to drastically reduce the number of freedom(10→2) per one unit for the structural analysis, an equivalent spring model is suggested as shown in Fig.12. The rotational spring:
K_θ is derived as follows. Refering to Fig.13,

$$K_\theta = \frac{6EI}{L} \quad \cdots\cdots(4)$$

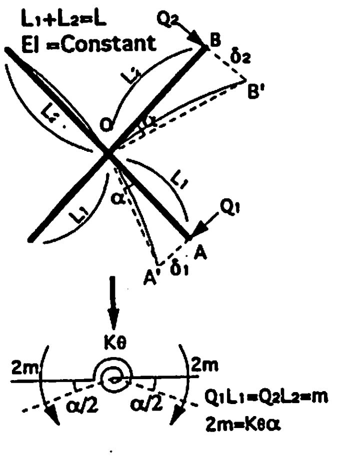


Fig.13 Evaluation of Kθ

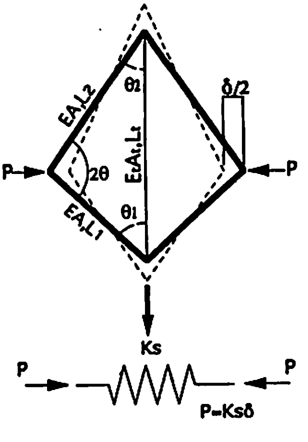


Fig.14 Evaluation of Ks

The axial spring:Ks is derived,refering to Fig.14.

$$K_s=0.5/(L_1/(EA(\sin\theta_1 \cos\theta_1 \tan\theta_2)^2)+L_2/(EA(\sin\theta_2+\cos\theta_2 \tan\theta_1)^2) +2L_T/(E_TA_T(\tan\theta_1+\tan\theta_2)^2) \quad \cdots(5)$$

And then,the basic equations of the spring structure shown in Fig.15,is derived from the principal of minimum potential energy. The potential energy :Π in this case is expressed as follows.

$$\Pi = 0.5 \sum_{i=1}^{n} K_{si}S_i^2 + 0.5 \sum_{i=2}^{n} K_{\theta i}(\alpha_i - \alpha_{(i-1)})^2 - \sum_{i=2}^{n} P_iV_i + \lambda_1 U_{n+1} \quad \cdots(6)$$

Where S_i, α_i, U_i and V_i are axial,angular distortion at i spring member,horizontal and vertical displacement at i point,respectively. λ_1 is a Lagrange's undetermined multiplier.

$$U_{n+1} = \sum_{i=1}^{n}(\,(l_i+S_i)\cos(\theta_i+\alpha_i) - l_i\cos\theta_i) \ (=0\ ;\ \text{boundary condition})$$

$$V_i = 0.5\,(\sum_{j=1}^{i-1}(l_j+S_j)\sin(\theta_j+\alpha_j) - \sum_{j=i}^{n}(l_j+S_j)\sin(\theta_j+\alpha_j))$$

l_i, θ_i are the length,the inclination angle of i spring member before loading respectively. Basic equations are derived by the stationary conditions concerned with S_i, α_i (i=1~n) and λ_1. And these equations are solved numerically by Newton−Raphson technique.

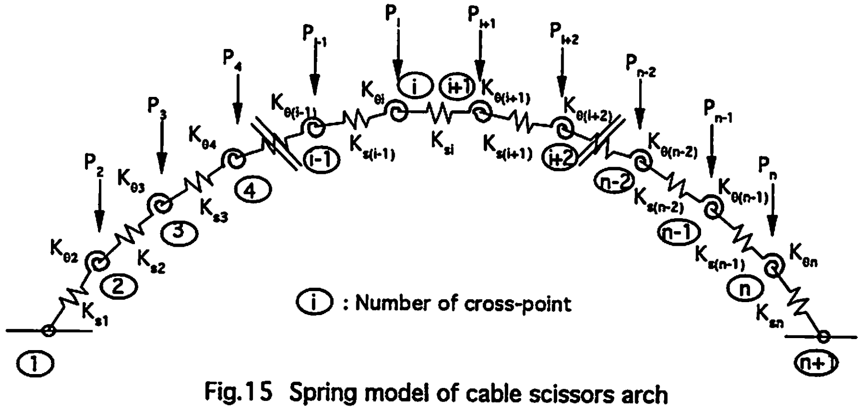


Fig.15 Spring model of cable scissors arch

4.3 Example of Numerical Analysis

Fig.16 shows a model for analysis. Applying to Eq.(4),(5), $K_{\theta i}$=2499 t·cm/rad, Ks=20.12 t/cm.

Fig.17 shows both the numerical results concerning vertical displacements and the axial forces.From these comparisons,it is shown that the suggested spring model gives a high approximate solution of frame model.

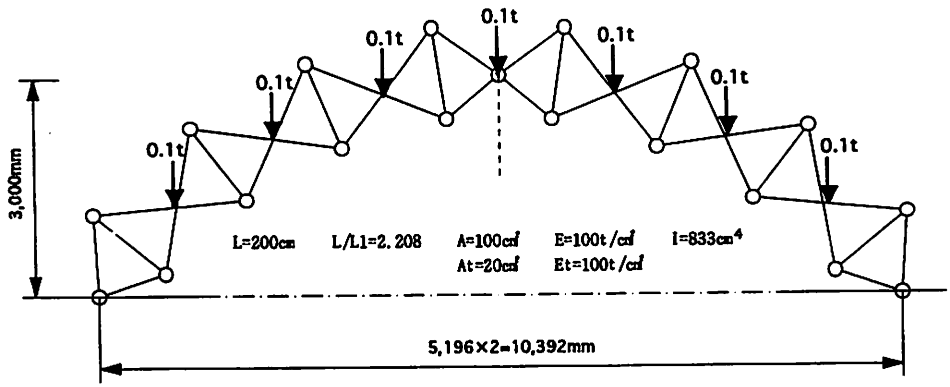


Fig.16 Model for numerical example

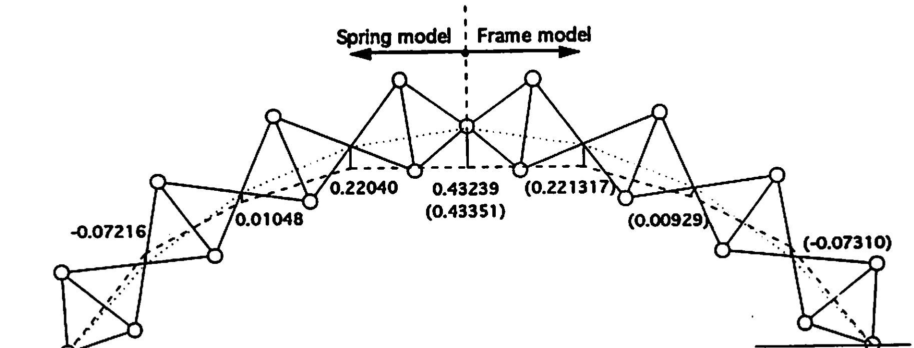


Fig.17a Vertical displacement (cm : linear analysis)

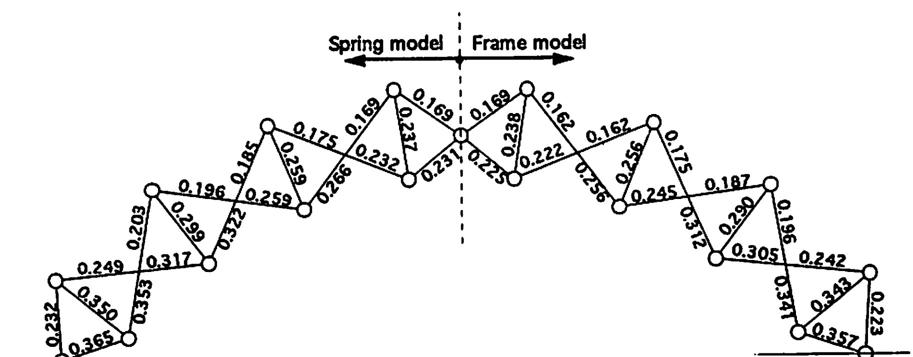


Fig.17b Axial force (ton : linear analysis)

5.CONCLUSION

Trial construction tests of a steel arch and a wooden dome with 5.3 m span,were carried out based on a simple idea about changing form structures,and we got a hopeful light of this method. Furthermore,the possibility of a spring model was shown as an approximate approach of the statics.

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